Pricing Options on High-Yield Corporate Bonds

M. Bank
M. Hanke
University of Innsbruck
Outline

1. Disclaimer
2. Problem description
3. Model
4. Results
5. Summary
Status

This talk describes “work at the beginning”, not (yet) “work in progress”. If we receive positive feedback, we will really start working on this project!
Pricing options on credit-risky bonds

- Many papers on reduced-form models for pricing credit-risky corporate securities (e.g., Duffie/Singleton (1999) and extensions, ...)
- Extensive literature on bond (option) pricing under interest rate risk
- Problem of bond option pricing in structural credit risk models has received little attention so far
- Argument for using this model class: Credit risk dominates interest rate risk for junk bonds ⇒ ignore interest rate risk and focus on credit risk
Possible reasons for lack of literature on bond pricing in structural models

- Well-known shortcomings of structural models
  - Credit spread often smaller than empirically observed spreads
  - PD ↓ 0 for short maturities

- Mathematical difficulties

- ?
Basis: Leland/Toft 1996

- Structural credit risk model featuring
  - finite-maturity, continuous-coupon debt
  - endogenous or exogenous, costly bankruptcy
  - stationary capital structure ⇒ constant default-triggering barrier \( L \)
  - taxes
  - deviations from absolute priority

- Value of assets \( (V) \) follows \( \frac{dV}{V} = (\mu - \beta)dt + \sigma dz \)
- Default occurs at \( \tau = \min_t : V_t = L \)
Model

Basis: Leland/Toft 1996

- Firm continuously issues new debt with maturity $T$ (and repays old debt) at a rate of $P/T$ per year
- Continuous coupon payments at a rate of $C$ per year
- Coupon is tax deductible
- Pre-specified fractions of firm value at default go to bankruptcy costs ($\varphi^K$), debt ($\varphi^D$), and (possibly) equity ($\varphi^E$)
Model

Basis: Leland/Toft 1996

- There exist closed-form solutions for
  - the value of debt
  - the value of equity
  - the value of the tax shield
  - the endogenous default level (optimal from equity’s view)
Modular pricing of options and corporate securities


- Payoffs to corporate securities are decomposed into their building blocks, which are priced separately
- Building blocks are then put together (“Lego-style”) to price securities (and options on them)
- Approach works well for many types of (exotic) barrier options
- Examples:
Modular pricing of options and corporate securities

Exponentially increasing unit down-and-in claim

\[ \Gamma_t^p(G_t) \]

Time

0 \quad \tau_1 \quad \tau_2 \quad \tau_3 \quad T
Modular pricing of options and corporate securities

Linearly decreasing down-and-out unit stream

\[ \Gamma_t^{\text{decU}} \]
Modular pricing of options and corporate securities

Application to the Leland/Toft model

E.g., current debt in the Leland/Toft framework can be valued as

\[
D(V, T, l_0, 0, C) = \frac{P}{T} U_{IO}(V, T, l_0, 0) + C \cdot \text{dec lin } U_{IO}(V, T, l_0, 0)
\]

\[
+ \varphi^D l_0 \cdot \text{dec lin } G^\rho_{ll}(V, T, l_0, 0).
\]
Model

Extension of Leland/Toft 1996: Hanke 2005

- Allows debt to increase exponentially
- Advantage: Avoids undesirable development of (expected) leverage
- Debt is allowed to increase at rate $\nu$
- Barrier also grows exponentially (starting value $l_0$, growth rate $\rho$, usually $\rho = \nu$)
- Features closed-form solutions for all corporate securities as well as options on equity
Options on “bond type 1”

- Underlying: “Chunk” of the bonds from the original capital structure, issued in interval of length $\varepsilon$
- Limitations:
  - Bond maturity restricted to interval $[0, T]$
  - Bond principal restricted to interval $[0, \varepsilon P / T]$
  - Equal seniority of all bonds (otherwise sort of “artificial”)
- Closed-form solution for standard options on this bond has been derived (no numerical results yet)
Options on “bond type 2”

- Underlying: Discrete-coupon bond, “on top of” the original capital structure
- Increased flexibility:
  - Bond maturity unrestricted
  - Bond principal unrestricted
  - Flexible seniority structure ($\phi$ for this bond may become very small or even zero if it is junior to the other bonds)
- Closed-form solution for standard options on this bond has been derived (no numerical results yet)
Summary

Open questions

- Current related work?
- Pursue this further? Different “base model”?
- …?
Results

Bond option pricing formulae

Call on bond type 1:

\[
C(D^U, K, s, l_0, \rho) = \alpha \varepsilon \left( \frac{P e^{\nu S}}{T} \mathcal{H}_{IO}(V, 0, S, l_0, \rho | A_s) \\
+ C_0 \exp(-\nu(S - T)) \mathcal{U}_{IO}(V, S, l_0, \rho | A_s) \\
+ \varphi^{DU} G^\rho_{ii}(V, S, l_0, \rho | A_s) \\
- e^{-rs} K \cdot \mathcal{H}_{IO}(V, \overline{V}, s, l_0, \rho) \right),
\]

where \( A_s = \{ V_s > \overline{V}, \tau \notin [0, s] \} \), and \( \overline{V} \) is defined by

\[
D^U(\overline{V}, S - s, \cdot) = K.
\]
Results

Bond option pricing formulae

Call on bond type 2:

\[ C(D^*, K, s, l_0, \rho) = P^* \mathcal{H}_{IO}(V, 0, S^*, l_0, \rho | A_s) \]
\[ + \sum_{i=[s]+1}^{[S^*]} c^* \mathcal{H}_{IO}(V, 0, t_i, l_0, \rho | A_s) \]
\[ + \varphi^* \mathcal{G}_{li}^\rho (V, S^*, l_0, \rho | A_s) \]
\[ - e^{-rs} K \cdot H_{IO}(V, \overline{V}, s, l_0, \rho), \]

where \( A_s = \{ V_s > \overline{V}, \tau \notin [0, s] \} \), and \( \overline{V} \) is defined by

\[ D^*(\overline{V}, S^* - s, \cdot) = K. \]